The Time Value of Money

- In order to work the problems in this module, the user should have the use of a business calculator such as the Hewlett Packard 17BII.
- The author grants individuals a limited license to use this presentation. It is the sole property of the author who holds the corresponding copyrights. The user agrees not to reproduce, duplicate or distribute any copies of this presentation in any form.
- The author would like to thank the Innovative Technology Center at The University of Tennessee which supported this project with a grant through the "Teaching with Technology Summer Institute." She would also like to commend the teachers who helped her design the module.
- If you have any comments or suggestions on how to improve this presentation, please e-mail the author at smurphy@utk.edu.

The Time Value of Money

- What is the "Time Value of Money"?
- Compound Interest
- Future Value
- Present Value
- Frequency of Compounding
- Annuities
- Multiple Cash Flows
- Bond Valuation

The Time Value of Money

Which would you rather have -- $1,000 today or $1,000 in 5 years?

Obviously, $1,000 today.

Money received sooner rather than later allows one to use the funds for investment or consumption purposes. This concept is referred to as the ***TIME VALUE OF MONEY!!***

Why TIME?

TIME allows one the *opportunity* to postpone consumption and earn INTEREST.

NOT having the opportunity to earn interest on money is called OPPORTUNITY COST.

How can one compare amounts in different time periods?

- One can adjust values from different time periods using an interest rate.
- Remember, one CANNOT compare numbers in different time periods without first adjusting them using an interest rate.

Compound Interest

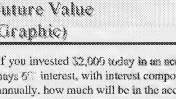
When interest is paid on not only the principal amount invested, but also on any previous interest earned, this is called compound interest.

FV = Principal + (Principal x Interest)
= 2000 + (2000 x .06)
= 2000 (1 + i)
= PV (1 + i)

Note: PV refers to Present Value or Principal

Future Value (Graphic)

If you invested $2,000 today in an account that pays 8% interest, with interest compounded annually, how much will be in the account at the end of two years if there are no withdrawals?



Future Value (Formula)

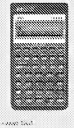
$FV_1 = PV(1+i)^n$ $= \$2{,}000\ (1.08)^2$
$= \$2{,}247.20$

FV = future value, a value at some future point in time
PV = present value, a value today which is usually designated as time 0
i = rate of interest per compounding period
n = number of compounding periods

Calculator Keystrokes: 1.08 (2nd) y^x 2 x 2000 =

Future Value (HP 17 B II Calculator)

Exit until you get Fin Menu.
2nd, Clear Data
Choose Fin, then TVM



Future Value Example

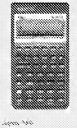
John wants to know how large his $5,000 deposit will become at an annual compound interest rate of 8% at the end of **5** years.

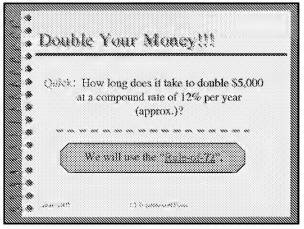
Future Value Solution

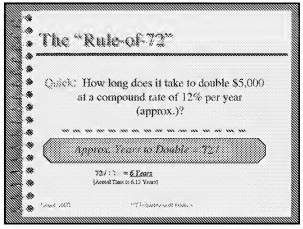
- Calculation based on general formula: $FV_n = PV(1+i)^n$
  $FV_5 = \$5{,}000\ (1+0.08)^5$
  $= \$7{,}346.64$
- Calculator keystrokes: 1.08 2nd y^x x 5000 =

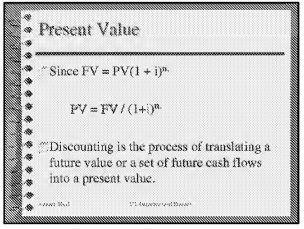
Future Value (HP 17 B II Calculator)

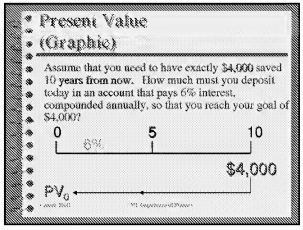
Exit until you get Fin Menu
2nd, Clear Data.
Choose Fin, then TVM

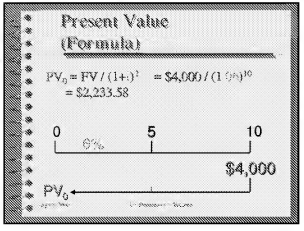


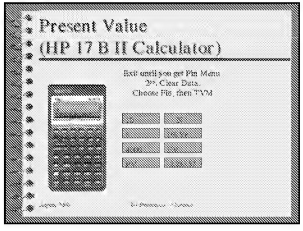

Double Your Money!!!
Quick: How long does it take to double $5,000 at a compound rate of 12% per year (approx.)?
We will use the "Rule-of-72".


The "Rule-of-72"
Quick: How long does it take to double $5,000 at a compound rate of 12% per year (approx.)?
Approx. Years to Double = 72/i
72/12 = 6 Years
(Actual Time is 6.12 Years)


Present Value
Since FV = PV(1 + i)n,
PV = FV / (1+i)n
Discounting is the process of translating a future value or a set of future cash flows into a present value.

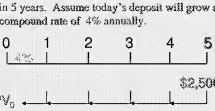

Present Value
(Graphic)
Assume that you need to have exactly $4,000 saved 10 years from now. How much must you deposit today in an account that pays 6% interest, compounded annually, so that you reach your goal of $4,000?
0
5
10
6%
$4,000
PV0


Present Value
(Formula)
PV0 = FV / (1+i)n = $4,000 / (1.06)10
= $2,233.58
0
5
10
6%
$4,000
PV0


Present Value
(HP 17 B II Calculator)
Exit until you get Fin Menu
2nd, Clear Data,
Choose Fin, then TVM

Present Value Example

Joann needs to know how large of a deposit to make today so that the money will grow to $2,500 in 5 years. Assume today's deposit will grow at a compound rate of 4% annually.

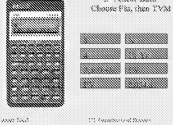


Present Value Solution

- Calculation based on general formula:
  $PV_0 = FV_n / (1+i)^n$
  $PV_0 = \$2,500/(1.04)^5$
  $= \$2,054.81$
- Calculator keystrokes: 1.04 2nd y^x 5 = 2nd 1/x X 2500 =

Present Value (HP 17 B II Calculator)

Exit until you get File Menu.
2nd, Clear Data
Choose File, then TVM

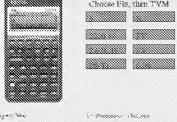


Finding "n" or "i" when one knows PV and FV

- If one invests $2,000 today and has accumulated $2,676.45 after exactly five years, what rate of annual compound interest was earned?

(HP 17 B II Calculator)

Exit until you get File Menu.
2nd, Clear Data
Choose File, then TVM



Frequency of Compounding

General Formula:

$$FV_n = PV_0(1 + [i/m])^{mn}$$

- n: Number of Years
- m: Compounding Periods per Year
- i: Annual Interest Rate
- FV_{end}: FV at the end of Year n
- PV_0: PV of the Cash Flow today

Frequency of Compounding Example

- Suppose you deposit $1,000 in an account that pays 12% interest, compounded quarterly. How much will be in the account after eight years if there are no withdrawals?

PV = $1,000
i = 12%/4 = 3% per quarter
n = 8 x 4 = 32 quarters

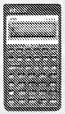
Solution based on formula:

$FV = PV(1 + i)^n$
$= 1{,}000(1.03)^{32}$
$= 2{,}575.10$

Calculator Keystrokes:
1.03 2nd y^x 32 X 1000 =

Future Value, Frequency of Compounding *(HP 17 B II Calculator)*

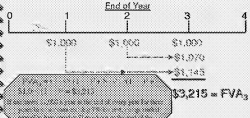
Exit until you get Fin Menu.
2nd, Clear Data
Choose Fin, then TVM



Annuities

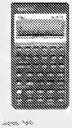
- *An Annuity* represents a series of equal payments (or receipts) occurring over a specified number of equidistant periods.
  - Examples of Annuities Include:
    - Student Loan Payments
    - Car Loan Payments
    - Insurance Premiums
    - Mortgage Payments
    - Retirement Savings

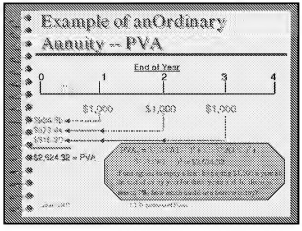
Example of an Ordinary Annuity -- FVA

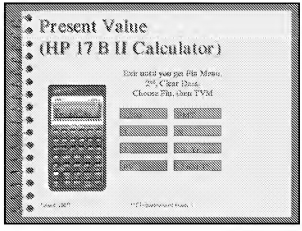


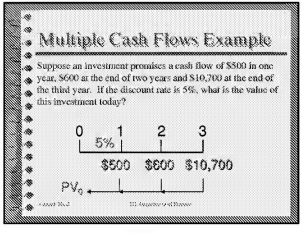
Future Value (HP 17 B II Calculator)

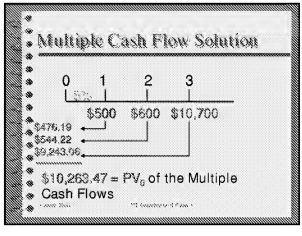
Exit until you get Fin Menu
2nd, Clear Data.
Choose Fin, then TVM

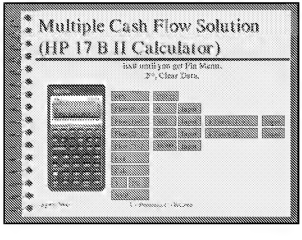


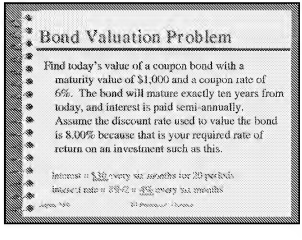

Example of anOrdinary Annuity -- PVA
End of Year
0
1
2
3
4
$1,000
$1,000
$1,000
$2,924.32 = PVA


Present Value (HP 17 B II Calculator)


Multiple Cash Flows Example
Suppose an investment promises a cash flow of $500 in one year, $600 at the end of two years and $10,700 at the end of the third year. If the discount rate is 5%, what is the value of this investment today?
0
1
2
3
5%
$500
$600
$10,700
PV0

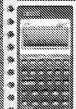

Multiple Cash Flow Solution
0
1
2
3
5%
$500
$600
$10,700
$476.19
$544.22
$9,243.06
$10,263.47 = PV0 of the Multiple Cash Flows


Multiple Cash Flow Solution (HP 17 B II Calculator)


Bond Valuation Problem
Find today's value of a coupon bond with a maturity value of $1,000 and a coupon rate of 6%. The bond will mature exactly ten years from today, and interest is paid semi-annually. Assume the discount rate used to value the bond is 8.00% because that is your required rate of return on an investment such as this.

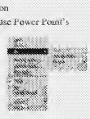
Bond Valuation Solution (HP 17 B II Calculator)

Exit until you get Fin Menu.
2nd, Clear Data



Welcome to the Interactive Exercises

- Choose a problem, select a solution
- To return to this page (slide 37), use Power Point's Navigation Menu
- Choose "Go" and "By Title"



Problem #1

You must decide between $25,000 in cash today or $30,000 in cash to be received two years from now. If you can earn 8% interest on your investments, which is the better deal?

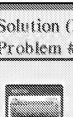
Possible Answers - Problem 1

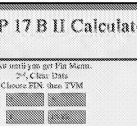
1. $25,000 in cash today
2. $30,000 in cash to be received two years from now
3. Either option O.K.

Need a Hint?

Solution (HP 17 B II Calculator) Problem #1

Exit until you get Fin Menu.
2nd, Clear Data
Choose FIN. then TVM





Compare PV of $30,000, which is $25,720.16 to PV of $25,000. $30,000 to be received 2 years from now is better.

Problem #2

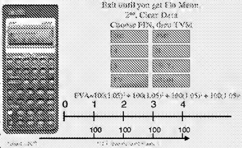
What is the value of $100 per year for four years, with the first cash flow one year from today, if one is earning 5% interest, compounded annually? Find the value of these cash flows four years from today.

Possible Answers - Problem 2

- $400
- $431.01
- $452.56

Need a Hint?

Solution (HP 17 B II Calculator) Problem #2



Problem #3

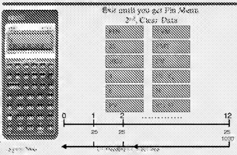
- What is today's value of a $1,000 face value bond with a 5% coupon rate (interest is paid semi-annually) which has three years remaining to maturity. The bond is priced to yield 8%.

Possible Solutions - Problem 3

- $1,000
- $921.37
- $1021.37

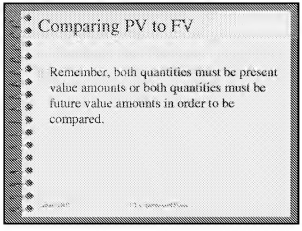
Need a Hint?

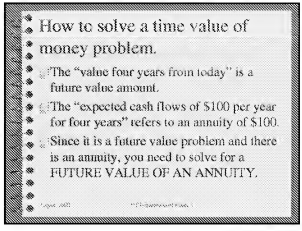
Solution (HP 17 B II Calculator) Problem #3

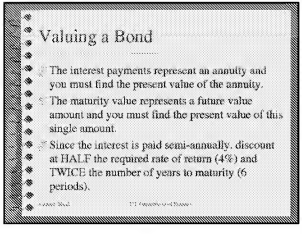


Congratulations!

- You obviously understand this material. Now try the next problem.
- The Interactive Exercises are found on slide #37.


Comparing PV to FV
Remember, both quantities must be present value amounts or both quantities must be future value amounts in order to be compared.


How to solve a time value of money problem.
The "value four years from today" is a future value amount.
The "expected cash flows of $100 per year for four years" refers to an annuity of $100.
Since it is a future value problem and there is an annuity, you need to solve for a FUTURE VALUE OF AN ANNUITY.


Valuing a Bond
The interest payments represent an annuity and you must find the present value of the annuity.
The maturity value represents a future value amount and you must find the present value of this single amount.
Since the interest is paid semi-annually, discount at HALF the required rate of return (4%) and TWICE the number of years to maturity (6 periods).